Cambridge Assessment International Education

# Cambridge International AS & A Level

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**MATHEMATICS** **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2024**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.




**[Turn over**

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**1** The coefficient of $x^2$ in the expansion of $(1-4x)^6$ is 12 times the coefficient of $x^2$ in the expansion of $(2+ax)^5$.

Find the value of the positive constant $a$. [3]

**2** The curve $y = x^2$ is transformed to the curve $y = 4(x-3)^2 - 8$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations have been applied. [5]

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**3** **(a)** Show that the equation $\frac{7\tan\theta}{\cos\theta}+12=0$ can be expressed as

$$12\sin^2\theta-7\sin\theta-12=0.$$ [3]

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**(b)** Hence solve the equation $\frac{7\tan\theta}{\cos\theta}+12=0$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]

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**4** The function f is defined as follows:

$$\mathrm{f}(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

**(a)** Find an expression for $\mathrm{f}^{-1}(x)$. [1]

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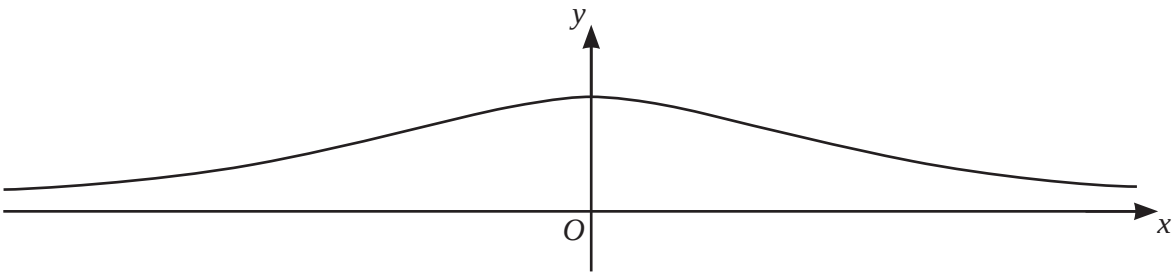


The diagram shows the graph of $y = \mathrm{g}(x)$ where $\mathrm{g}(x) = \dfrac{1}{x^2 + 2}$ for $x \in \mathbb{R}$.

**(b)** State the range of g and explain whether $\mathrm{g}^{-1}$ exists. [2]

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The function h is defined by $\mathrm{h}(x) = \dfrac{1}{x^2+2}$ for $x \geqslant 0$.

**(c)** Solve the equation $\mathrm{hf}(x) = \mathrm{f}\left(\dfrac{25}{16}\right)$. Give your answer in the form $a + b\sqrt{c}$, where $a$, $b$ and $c$ are integers. [4]

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**5** The first and second terms of an arithmetic progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

**(a)** Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

The first and second terms of a geometric progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

**(b) (i)** Find the sum to infinity of the progression in terms of $\theta$. [2]

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**(ii)** Given that $\theta = \frac{1}{3}\pi$, find the sum of the first 10 terms of the progression. Give your answer correct to 3 significant figures. [3]

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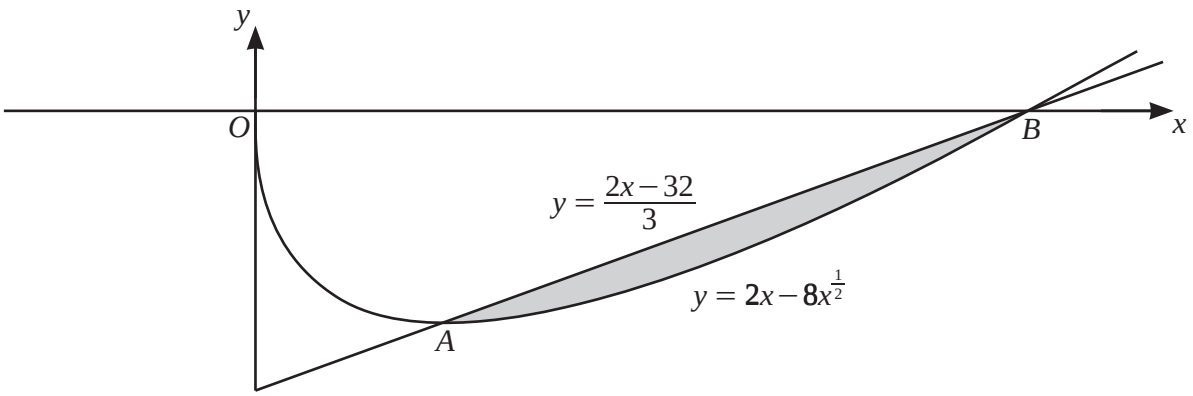
**6** The curve with equation $y = 2x - 8x^{\frac{1}{2}}$ has a minimum point at $A$ and intersects the positive $x$-axis at $B$.

**(a)** Find the coordinates of $A$ and $B$. [4]

**(b)**




The diagram shows the curve with equation $y = 2x - 8x^{\frac{1}{2}}$ and the line $AB$. It is given that the equation of $AB$ is $y = \frac{2x-32}{3}$.

Find the area of the shaded region between the curve and the line. [5]

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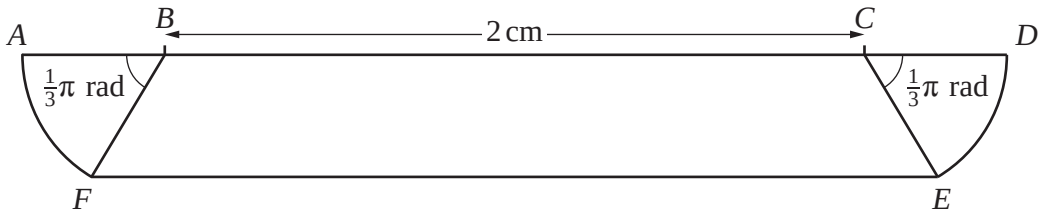
**7** The equation of a circle is $(x-6)^2+(y+a)^2=18$. The line with equation $y=2a-x$ is a tangent to the circle.

**(a)** Find the two possible values of the constant $a$. [5]

**(b)** For the greater value of $a$, find the equation of the diameter which is perpendicular to the given tangent. [3]

**8**




The diagram shows a symmetrical plate *ABCDEF*. The line *ABCD* is straight and the length of *BC* is 2 cm. Each of the two sectors *ABF* and *DCE* is of radius $r$ cm and each of the angles *ABF* and *DCE* is equal to $\frac{1}{3}\pi$ radians.

**(a)** It is given that $r = 0.4$ cm.

**(i)** Show that the length $EF = 2.4$ cm. [2]

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**(ii)** Find the area of the plate. Give your answer correct to 3 significant figures. [4]

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**(b)** It is given instead that the perimeter of the plate is 6 cm.

Find the value of $r$. Give your answer correct to 3 significant figures. [4]

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**9** A function f is such that $f'(x) = 6(2x-3)^2 - 6x$ for $x \in \mathbb{R}$.

**(a)** Determine the set of values of $x$ for which $f(x)$ is decreasing. [4]

**(b)** Given that $f(1) = -1$, find $f(x)$. [4]

**10** The equation of a curve is $y = (5-2x)^{\frac{3}{2}} + 5$ for $x < \frac{5}{2}$.

**(a)** A point $P$ is moving along the curve in such a way that the $y$-coordinate of point $P$ is decreasing at 5 units per second.

Find the rate at which the $x$-coordinate of point $P$ is increasing when $y = 32$. [4]

**(b)** Point $A$ on the curve has $y$-coordinate 32. Point $B$ on the curve is such that the gradient of the curve at $B$ is $-3$.

Find the equation of the perpendicular bisector of $AB$. Give your answer in the form $ax+by+c=0$, where $a$, $b$ and $c$ are integers. [6]

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## Additional page

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